The Primordial Density Perturbation Supplementary Material and Further Developments

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In Section 1 we give supplementary material, that may help readers who encounter a difficulty when reading the text. Please let us know of any other queries about the text.

In Section 2 we go beyond the text, covering mostly recent developments.

The references should help the reader to access further literature. They don't necessarily indicate the originators of a line of research.

1. Supplementary Material

Chapter 2

**** 2.6.1*

To define the number density of particles, appearing in Eq. (2.57), we have to consider a small region of space, centred at $\mathbf{r}$, that contains many particles and yet is small compared with the distance scales over which we want to use Eq. (2.57). The quantum theory of these particles is to be formulated within such a region (to be precise, within a fictional box that is somewhat bigger so that the discreteness of the momentum is negligible).

Chapter 3

**** Section 3.2*

The last paragraph of this section, commenting on the use of a locally inertial frame, is not very appropriate at that point because in flat spacetime we can extend the locally inertial frame indefinitely (corresponding simply to an inertial frame). The paragraph should really go at the end of Section 3.5 where curved spacetime is considered. The curvature scale is the maximum distance over which $g_{\mu\nu}$ and $\partial_\lambda g_{\mu\nu}$ can be chosen to be nearly constant, ie. the maximum size of a region within which the locally inertial frame is useful.

Chapter 5

**** 5.2.2*

The issue of smoothing in general relativity is addressed in S. R. Green and R. M. Wald, arXiv:1011.4920 [gr-qc].

**** 5.4.1*

The heading should really be before the previous paragraph because the super-horizon smoothing is a crucial part of the definition of ζ.

**** 5.4.3*

In Section 5.4.3 it's only the slicing that is supposed to be different from Section 5.4.1, the threading is still supposed to be comoving. We're denoting the time coordinate of Section 5.4.3 by the same symbol t as the time coordinate t of Section 5.4.1, but one is actually a function of the other because the slicing is different. If we denote the time coordinate of Section 5.4.3 by $\tilde{t}$, we have simply $\tilde{a}(\tilde{\mathbf{x}}, \tilde{t}) = a(\mathbf{x}, t)$, because the new and old coordinates label the same point in spacetime. Then Eq. (5.17) applies with $f = \ln a$, leading immediately to Eq. (5.22).

As stated in footnote 2, Eq. (5.22) can also be derived from Eq. (5.21), and that derivation remains valid if the definitions of ψ and ζ are extended so as to apply without any smoothing (and, as it turns out, without specifying any threading). But it's not very useful to make that extension.

Chapter 8

On the right hand sides of Eqs. (8.34) and (8.47), the first two terms can be combined to give $-aH(3c_{\rm s}^2 - 1)$, where $c_{\rm s}^2 \equiv \dot{P}/\dot{\rho}$. This helps to prove Eq. (8.57). (As we have noted in the errata file, $\dot{R}$ in that equation should be R.)

Chapter 10

10.7 Acoustic peaks and Silk damping

In this section we estimate the first and second terms of Eq. 10.41, called respectively monopole and dipole terms. We find dipole is fairly smooth so that the peak structure of C_ℓ comes mainly from the monopole. This is nicely confirmed in Figure 1 below, taken from A. Challinor and H. Peiris, arXiv:0903.5158 [astro-ph.CO]. The red line is the monopole while the blue line is the dipole. The green line is the integrated Sachs-Wolfe (ISW) effect, corresponding to the last term of Eq. 10.37 whose contribution to C_ℓ is given by the second line of Eq. 11.76. The ISW effect is small and the contribution of the term proportional to P in Eq. (11.76) (not shown) is even smaller so that the monopole and dipole dominate in accordance with our discussion.

Chapter 13

*** In the third paragraph of page 218, '4-divergence' has not been defined. The 4-divergence of a 4-vector A^μ is $\partial_\mu A^\mu$, and its integral over a spacetime volume vanishes if A^μ vanishes on the boundary.

*** To do Exercise 13.4 you will have to use the definitions (2.39) and (2.40) of energy density and pressure, which are valid in a local rest frame.

Chapter 14

*** The λ^2 of Eq. (14.53) is the $\lambda/2$ of Eq. (14.32), and the $v^2/2$ of Eq. (14.53) is the f^2 of Eq. (14.32). These conventions simplify the subsequent discussion.

Chapter 15

*** The second of the equations labeled (15.27), stating that the annihilation operators of *different* harmonic oscillators commute, is very important for the rest of the chapter and should perhaps have been stated explicitly. Thus, for a single free scalar field $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = 0$, and for the operators $a_{n\mathbf{k}}$ belonging to real free scalar fields $\phi_1, \phi_2, \cdots$ we have $[\hat{a}_{n\mathbf{k}}, \hat{a}_{m\mathbf{k}'}] = 0$ and $L^{-3}[\hat{a}_{n\mathbf{k}}, \hat{a}^\dagger_{m\mathbf{k}'}] = \delta_{nm}\delta_{\mathbf{k}\mathbf{k}'}$. The latter case, with two real fields, leads to Eq. (15.41) for a single complex field, with the additional condition $[\hat{a}(\mathbf{k}), \hat{b}(\mathbf{k})] = [\hat{a}(\mathbf{k}), \hat{b}^\dagger(\mathbf{k})] = 0$.

*** After Eq. (15.62) we find that the late-time expectation value, of the occupation number of the particles corresponding to ϕ, is equal to $|\beta_k|^2$. This corresponds to the existence of a gas of these particles, whose occupation number averaged over a cell d^3k centered on on $\mathbf{k}$ is $|\beta_k|^2$. In other words, $|\beta_k|^2$ is the distribution function $f_{\rm a}(\mathbf{k})$ considered in Eq. (2.57).

Chapter 18

*** The statement on page 308, that $aH \ll H_0$ is sufficient to make the present value of Ω close to 1, implicitely assumes that the initial value of $|1-\Omega|$ is not much bigger than 1. That is a reasonable assumption in the context of present ideas about what might have happened before inflation. In particular, tunneling from a metastable vacuum could give an initial Ω very close to 0 at the beginning of inflation (M. Bucher, A. S. Goldhaber, N. Turok (Phys. Rev. **D52**, 3314-3337 (1995)). But the tunneling setup doesn't guarantee that $1-\Omega_0$ will be too small to ever observe. Versions of the setup allowing an observable $1-\Omega_0$ as well as a non-standard tensor perturbation are considered by D. Yamauchi, A. Linde, A. Naruko, M. Sasaki and T. Tanaka (arXiv:1105.2674 [hep-th]).

*** After Eq. (18.29) we deduce that all sufficiently-nearby solutions $H(\phi)$ converge to a common solution. The desired result, that the same is true of $\phi(t)$ up to a time translation, then follows from Eq. 18.21).

*** Here's the proof that the condition given three lines after Eq. (18.29) follows from that equation. With that equation, the condition is equivalent to $d(He^N)/dt > 0$ (where H is the unperturbed quantity denoted in the text by H_0) which is just the condition Eq. (18.3) for inflation.

*** The hybrid inflation critical value $\phi_{\rm c} = m_\chi/\sqrt{\lambda'}$ is calculated by setting $\partial^2 V/\partial\chi^2$ equal to zero at $\chi = 0$.

*** In Ex. 18.3, the proof of Eq. (18.8) is independent of Ω_Λ because all relevant quantities are evaluated well before Ω breaks away from 1. In contrast, one needs to know Ω_Λ to evaluate the redshift at which k_0 enters the horizon.

Chapter 20

*** 20.2.1 Light fields

In the context of Einstein gravity, the definition of light fields, specifying flatness conditions on the potential, is assumed to lead to the slow-roll conditions (20.3) and (20.4). Also, the definition of light fields without a gravity theory is assumed to lead to (20.4) (it being understood that (20.3) is supposed to hold). These assumptions are reasonable for the reason explained after Eq. (20.10). The idea here is to liberate the notion of a light field from the notion of multi-field inflation. (In the Einstein gravity case, this corresponds to allowing $V \ll \rho$ during inflation, with the dominant contribution to ρ coming from some other source of gravity.) The generation of light field perturbations (Chapter 24) and the generation of ζ through curvaton-type mechanisms (Chapter 26) then becomes independent of the inflation mechanism, though of course that mechanism must be such that the quantum theory of the scalar fields is viable.

Chapter 24

*** In Section 24.3 we simplify the notation by dropping without comment the the subscript $\mathbf{k}$ on the Fourier component $\delta\phi_{\mathbf{k}}$. For φ, we write $\varphi(\mathbf{k})$ for the Fourier component and $\varphi(k)$ for the mode function, where we have previously been writing $\varphi_{\mathbf{k}}$ and φ_k.

The following comments about Eqs. (24.29)-(24.31) may be helpful. Their right hand sides, corresponding to back-reaction, will clearly involve only the scalar mode of the metric perturbations. The flat slicing, on which Eqs. (24.30) and (24.31) apply, is defined after Eq. (5.14). Equivalently, as we are working to first order, it is defined as the one with $D = 0$ where D is defined by Eq. (8.8). The condition $D = 0$ indeed defines a unique slicing, because Eq. (8.27) says that the remaining spatial metric perturbation E can be removed by a purely spatial coordinate transformation. Such a transformation has no effect on the scalar field perturbation, whose transformation is given by Eq. (5.19).

Setting both D and E to zero on the flat slicing, and transforming to a generic gauge using Eqs.(5.19), (8.26) and (8.27), one finds

$$\delta\phi = \widetilde{\delta\phi} + \frac{\dot\phi}{H}\left(D + E/3\right). \qquad (1)$$

The right hand side may be regarded as a gauge-invariant field perturbation.

To arrive at Eq. (24.30), one should work out the right hand side of Eq. (24.29)) in terms of the metric perturbations using the compact expression (3.22), and then relate the metric perturbations to the field perturbation using the Einstein equation. These manipulations are most simply done in the conformal Newtonian gauge, after which one can use Eq. (1) to arrive at the flat slicing. Note that anisotropic stress vanishes because the energy-momentum tensor comes entirely from scalar fields, which means that $\Phi = \Psi$. [end of comments]

Going to the multi-field case, Eq. (24.30) applies at each instant in a field basis where ϕ is the only field with nonzero $\dot\phi$. Rotating to a time-independent basis one then immediately obtains Eq. (24.31).

Chapter 25

25.7 K-inflation

Following the most usual nomenclature, we are taking 'k-inflation' to mean a model using the lagrangian density (25.49) where the vacuum does *not* spontaneously break Lorentz invariance. In the opposite case the vacuum can have $\dot\phi$ equal to a nonzero constant [N. Arkani-Hamed, H.-C. Cheng, M. A. Luty and S. Mukohyamam, JHEP **0405** (2004) 074]. Then Lorentz invariance is spontaneously broken and there is an apparent modification of Einstein gravity *** corresponding to what is called ghost inflation. The general form of the action described in [20] includes both of these possibilities, and others. For more work in that direction see S. Weinberg, Phys. Rev. D **77** (2008) 123541.

25.8 Warm inflation

*** For a review of warm inflation see A. Berera, I. G. Moss and R. O. Ramos, Rept. Prog. Phys. **72** (2009) 026901. It typically gives non-gaussianity at an observable level (I. G. Moss and T. Yeomans, arXiv:1102.2833).

Chapter 26

*** *Formula for the spectral index*

Eq. (26.7) with $V/M_{\rm Pl}^2$ replaced by $3H^2$ follows just from Eq. (20.4) at each location. The latter equation is the slow-roll approximation to the exact unperturbed Eq. (13.58), which is valid at each location provided that the fields ϕ_i are smoothed on the scale corresponding to the horizon at horizon exit. By invoking such smoothing to the fields we drop some structure that might in principle contribute to the smoothed ρ and p that are needed to calculate N but such a contribution is usually negligible in practice.

But to get the right hand side of Eq. (26.8), and hence the desired result (26.6) for the spectral index, we need $V/M_{\rm Pl}^2$ which invokes Einstein gravity. Therefore, Eq. (26.6) is entirely equivalent to Eq. (26.10). However, the only role played by the right hand side of Eq. (26.8) is to generate the final term of Eq. (26.6). The rest of the expression, which typically dominates, depends only on the slow roll approximation (20.4) and is independent of both the theory of gravity and the mechanism of inflation.

****Evolution of the curvaton field*

A contribution to the curvature perturbation, that is generated at some epoch after inflation, will exist only on scales that are outside the horizon at that epoch. To calculate such a contribution we should smooth the energy density and pressure on a super-horizon scale, which means that we should smooth the curvaton field on that scale.[*] Its evolution after relevant scales leave the horizon, represented by the function g, is at each location is the same as for an unperturbed universe (the separate universe assumption, valid if the longest relevant scale for the evolution is much smaller than the smoothing scale). In other words, $\chi(\mathbf{x},t)$ satisfies Eq. (13.57) with $\phi\equiv\chi$.

****Curvaton decay rate*

The mechanism explained at the bottom of page 352 makes the decay rate of the oscillating thermal inflation field much smaller than the decay rate of the corresponding particle. The same mechanism can apply to any oscillating field, in particular to the curvaton (K. Enqvist, A. Mazumdar and O. Taanila, JCAP **1009** (2010) 030). If the curvaton potential is quadratic, this dramatically strengthens the bound $H_*\gtrsim 10^7\,\text{GeV}$, to become $H_*\gtrsim 10^{10}\,\text{GeV}$ (C. S. Chen, L. Y. Lee and C. M. Lin, arXiv:1105.3801).

****Proof of Eqs (26.25) and (26.29)*

In the line after Eq. (26.24), the function $N(g(\sigma_*),\rho_1,\rho_2)$ is simply $(1/4)\ln(\rho_1/\rho_{\rm rad})$, because $\rho_{\rm rad}$ is proportional to a^{-4}. In turn, $\rho_{\rm rad}(g(\sigma_*),\rho_1,\rho_2)$ is given by Eq. (26.24), after eliminating ρ_σ. The calculation of the partial derivatives N_σ and $N_{\sigma\sigma}$, at fixed ρ_1 and ρ_2, is then a straightforward exercise in partial differentiation.

**** Proof of Eq (26.26)*

Use $\rho_{\rm dec}\simeq T^4_{\rm dec}$ and $\rho_1=3M^2_{\rm Pl}H^2$ with $H\simeq m_\sigma$.

****Inclusion of the inflaton contribution to ζ*

In the text we assume that the curvaton contribution to ζ is dominant, so that ζ is negligible when the curvaton oscillation starts. In fact, the formulas give the curvaton contribution to ζ even if the inflaton (or some other) contribution makes ζ significant when the oscillation starts. This follows from Eq. (5.15), according to which $\zeta(\mathbf{x},t)$ is the perturbation in the number of e-folds from an arbitrary flat slice, to a slice of uniform ρ at time t. It follows that the *difference* $\zeta(\mathbf{x},t_2)-\zeta(\mathbf{x},t_1)$ between two epochs is equal to the perturbation in the number of e-folds between slices of uniform ρ, which is the quantity that we evaluate in the text. Similarly, the formulas for the inhomogeneous decay rate case apply even if ζ is significant just before the decay.

****End of inflation contribution to ζ*

The end of inflation in (26.41) was given in D. H. Lyth, JCAP **0511** (2005) 006 (see F. Bernardeau, L. Kofman and J. P. Uzan, Phys. Rev. D **70** (2004) 083004 for an equivalent expression to first order in the field perturbations). It holds only if the transition from slow-roll inflation to non-inflation is sudden, ie. the time taken is much less that the displacement δt between the uniform density slice and the end-of-inflation slice. This should be checked case by case, which has not been done in the literature.

****Contribution to ζ from preheating*

The contribution to ζ generated during the linear era of the waterfall of hybrid inflation (tachyonic preheating) has $\mathcal{P}_\zeta\propto k^3$ (D. H. Lyth, arXiv:1012.4617 []). The same is true for the contribution generated during the linear era of ordinary preheating, with inflaton potential $V\propto\phi^2$ (A. R. Liddle, D. H. Lyth, K. A. Malik and D. Wands, Phys. Rev. D **61** (2000) 103509) and with most of the usually-considered inflation potentials. The k^3 dependence in these examples occurs because the relevant field χ is heavy, and is probably a general result for the contribution to ζ generated by any heavy field. With such a dependence, the black hole bound on the scale leaving the horizon at the end of inflation makes $\mathcal{P}_\zeta$ negligible on cosmological scales.

With the inflation potential $V\propto\phi^4$, it is possible for the χ of ordinary preheating to be light. Then its contribution to $\mathcal{P}_\zeta$ can be nearly scale-independent, and contribute on cosmological scales. That case is called massless preheating for which we cite [10]. But the problem of calculating the contribution to ζ in that case is actually very difficult, and so far unsolved (K. Kohri, D. H. Lyth and C. A. Valenzuela-Toledo, JCAP **1002** (2010) 023)). Preheating of any kind can give a nearly flat contribution to ζ, if some coupling or mass involved depends on the value of a light field. This 'inhomogeneous preheating' is considered in the previous reference.

[*] Strictly speaking that is true only for the contribution to ζ that is linear in the curvaton field perturbation, ie. for the gaussian part of ζ, but the contribution of sub-horizon modes is anyhow negligible even discounting their attenuation by redshift as described in Section 27.2 for the analogous case of the axion.

Reference [12]

An earlier paper along the same lines is J. McDonald, Phys. Rev. D **69** (2004) 103511.

Chapter 27

27.2.2 A non-gaussian axion isocurvature perturbation

A fuller version of our Figure 27.1 is given in Figure 2, taken from J. Hamann, S. Hannestad, G. G. Raffelt and Y. Y. Y. Wong, arXiv:0904.0647 [hep-ph]. Compared with our Figure 27.1, the axes of Figure 2 are interchanged and $V^{1/4}$ is replaced by $H = \sqrt{V/3M_{\rm Pl}^2}$.

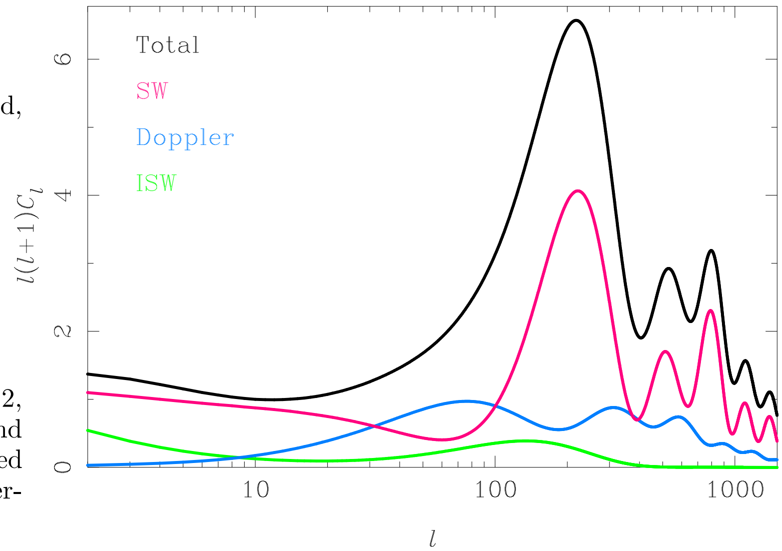

FIG. 1: Contribution to C_ℓ, in an arbitrary unit.

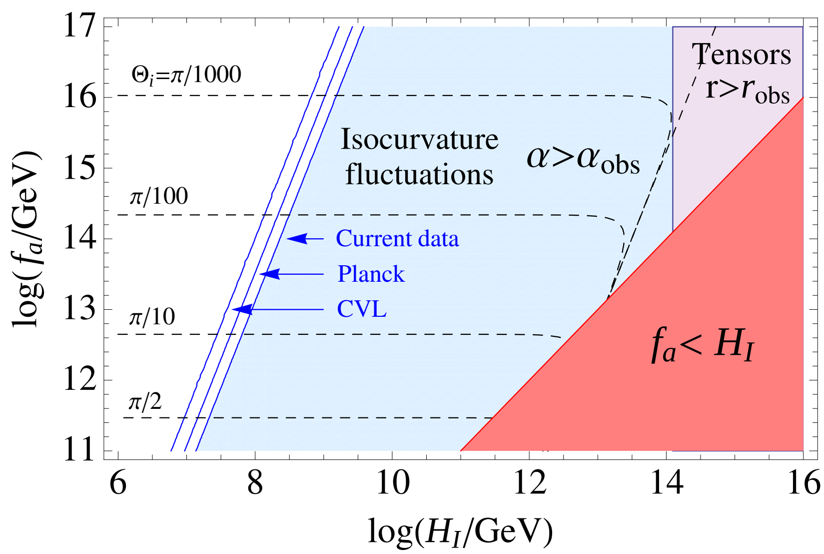

FIG. 2: Exclusion and sensitivity regions in the plane of H_I (Hubble rate during inflation) and $f_{\rm a}$ (axion decay constant), assuming axions are all of the dark matter. The isocurvature exclusion region based on current data is shown in light blue. The sensitivity forecasts for Planck and CVL are also indicated. The dashed lines indicate the required $\Theta_{\rm i}$ for a given $f_{\rm a}$ to obtain the full amount of axion dark matter. Also shown is the region of excessive tensor modes and the region $f_{\rm a} < H_I$ corresponding to the string regime.

2. Further developments

PART I

**** Complete second order calculation*

A complete second-order version of the calculation described in Chapter 11 is given for the first time by C. Pitrou, J. P. Uzan and F. Bernardeau, "The cosmic microwave background bispectrum from the non-linear evolution of the cosmological perturbations," JCAP **1007** (2010) 003.

PART II

**** Statistical inhomogeneity and statistical anisotropy*

On page 86 we discussed the assumptions of translation invariance and rotational invariance, for the statistical properties of a perturbation. These are usually called, respectively *statistical homogeneity* and *statistical isotropy*. Current observational bounds on statistical inhomogeneity and statistical anisotropy are both at very roughly the 10% level. As seen below, the latter might easily be generated within the inflationary scenario.

PART III

Gauge mediated supersymmetry breaking with 100 GeV *gravitino mass*

In our discussion of gauge mediated supersymmetry breaking, we implicitly assumed that gravity mediated supersymmetry breaking would always be present. To allow the gauge mediation mechanism to dominate one then requires the supersymmetry breaking scale to be well below 100 GeV as we stated, corresponding to a gravitino mass well below 100 GeV. Such a conclusion has the important consequence that CDM cannot be the neutralino, since that particle would decay into the lighter gravitino.

Instead of the above scenario, it could be that the gravity mediated supersymmetry breaking is suppressed so that the gauge mediated supersymmetry breaking can be the dominant mechanism even with a gravitino mass of order 100 GeV. Such a scenario has the best of both worlds; (i) it keeps the pattern of supersymmetry breaking under control because it is gauge mediated, and (ii) it allows the neutralino to be the CDM and it naturally generates the required μ and $B\mu$ terms in the MSSM because the symmetry breaking scale is the same as it would be for gravity mediation.

This having being said, it remains true that the first scenario of gauge mediation is the most generic and the second scenario has yet to be realized within the simple gauge mediation schemes that invoke a metastable vacuum.

PART IV

Supersymmetric axion physics

**** Axionic dark matter*

In the misalignment scenario (as opposed to the axionic string scenario) axionic dark matter consists of an oscillating scalar field. As a result, the distribution of CDM in galaxy halos will be different, in a way that may be confirmed or ruled out by future observation (P. Sikivie, arXiv:1012.1553 [astro-ph.CO]).

Saxion dark matter

A CDM candidate that we didn't mention is the saxion. It isn't a supersymmetric particle but its lifetime is bigger than the age of Universe if it is very light, roughly $\lesssim 10\,\text{keV}$. As the saxino mass is typically of order the gravitino mass this corresponds to a gravitino mass of the same order. Depending on the parameters and the cosmology, the saxion might be either CDM or warm dark matter.

When the saxion is in thermal equilibrium, the minimum of its effective potential is displaced from the origin and the reheat temperature [K. Nakayama and F. Takahashi, Phys. Lett. B **670** (2009) 434]. This happens because the saxion is a PNGB which means that its vev doesn't correspond to a fixed point of a symmetry. Demanding that the present saxion density is not too big requires a reheat temperature $\lesssim 10^3\,\text{GeV}$ for a stable saxion. This constraint is very roughly similar to the bound on the reheat temperature for a gravitino or axino of similar mass.

Evolution of the curvaton field

The linear evolution mentioned two lines before Eq. (26.33) corresponds to a quadratic potential. Small deviations from the quadratic form can have a strong effect (K. Enqvist, arXiv:1012.1711). Assuming an approximately quadratic form, the effect of preheating caused by the curvaton oscillation has been considered (K. Enqvist, S. Nurmi and G. I. Rigopoulos, JCAP **0810** (2008) 013).

An entirely different possibility is for the curvaton to be near a maximum of its potential during inflation, as discussed in the last paragraph of Section 26.4.2. Some further aspects of this 'hilltop non-gaussianity' are discussed by M. Kawasaki, K. Nakayama and F. Takahashi (JCAP **0901** (2009) 026).

*** Statistical anisotropy and statistical inhomogeneity

Statistical anisotropy

For the two-point correlator of ζ, the leading signal of statistical anisotropy will be of the form

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k)\left(1 + g(\mathbf{a}\cdot\mathbf{k})^2/k^2\right)$$

where $\mathbf{a}$ is a unit vector in some preferred direction. This would give the two-point correlator of the cmb temperature, seen in a small patch of sky, a quadrupole dependence on the location of that patch. Statistical anisotropy for higher correlators of may also exist, and could me more important than for the spectrum.

Several papers have been published on mechanisms for making the primordial curvature perturbation ζ statistically anisotropic. Most of them invoke vector fields.

One possibility is for a homogeneous vector field to generate anisotropic inflation, with ζ coming as usual from the perturbation of a scalar field. This will also affect the primordial tensor perturbation, causing it to be polarized and correlated with ζ. To implement this scenario, the gauge kinetic function of the vector field should have a suitable time-dependence during inflation, coming presumably from coupling to the inflaton field (M. a. Watanabe, S. Kanno and J. Soda, arXiv:1011.3604)).

A different possibility is for the perturbation of one or more vector fields to give a contribution to ζ, generated during practically isotropic inflation. Within a curvaton-type mechanism, this is likely to generate observable non-gaussianity, whose statistical anisotropy will be a smoking gun for the scenario (K. Dimopoulos, M. Karciauskas, D. H. Lyth and Y. Rodriguez, JCAP **0905** (2009) 013). To avoid too much anisotropy, the contribution of the vector field perturbation to ζ should generally be subdominant, or else many vector fields should contribute. To implement this scenario, either the gauge kinetic function should have suitable time-dependence, or else (M. Karciauskas and D. H. Lyth, JCAP **1011** (2010) 023) it should have a coupling to gravity of the form $R^2A^2/6$.

Either of the scenarios might work with a non-Abelian gauge field (K. Murata and J. Soda, 1003:6164; N. Bartolo, E. Dimastrogiovanni, S. Matarrese and A. Riotto, JCAP **0910** (2009) 015).

**** Statistical inhomogeneity*

A signal for statistical inhomogeneity would be a *dipole* dependence of the cmb multipoles seen in a small patch of sky (as opposed to the quadrupole generated by statistical anisotropy). Enough inflation makes the Universe absolutely homogeneous and isotropic at the classical level. All perturbations must then originate as vacuum fluctuations, and since the vacuum is translation invariant the perturbations will be statistically homogeneous. Of course, this statistical homogeneity refers to the ensemble of all possible perturbations (all possible outcomes of a measurement, within the standard interpretation of quantum physics). If the observable Universe is at a very untypical location, the observed perturbations might appear to be statistical inhomogeneous. (S. M. Carroll, C. Y. Tseng and M. B. Wise, Phys. Rev. D **81** (2010) 083501.)

*** First-order QCD phase transition

On page 345 we mention two possible effects of a first-order QCD phase transition, namely the formation of light CDM halos and the generation of a small-scale baryon isocurvature perturbation affecting BBN. It is now known from lattice simulations that the transition is *not* of first order if the baryon and lepton number densities are very small, corresponding to negligible chemical potentials.. We know that at least the baryon number density is very small at present, but it might have been big (corresponding to a significant baryon chemical potential) before the QCD phase transition. In that case the QCD transition might indeed have been first order, giving rise to several possible effects in addition to those mentioned on page 345 (T. Boeckel and J. Schaffner-Bielich, arXiv:1105.0832).

The most dramatic effect is a few e-folds of thermal inflation. This would provide a mechanism (actually the only feasable one at such a late epoch) for reducing the baryon number number density to its present level. It would also dilute the CDM density, allowing CDM domination before the QCD transition. Other effects are the generation of a primordial magnetic field and the generation of primordial gravitational waves.

QCD thermal inflation

Slow-roll Inflation models

****Feature in the potential*

During single-field slow-roll inflation, a feature in the potential may cause a temporary failure of slow-roll. This may cause a feature in the spectrum of the curvature perturbation [J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64** (2001) 123514] and may also generated observable non-gaussianity [J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64** (2001) 123514].

****Inflation from monodromy*

String theory suggests the existence of trajectories in the space of the scalar fields, that can wind many times around the origin. This ‘monodromy’ provides a way of building large-field ‘chaotic inflation’ types of potential (E. Silverstein and A. Westphal, Phys. Rev. D **78** (2008) 106003). The potential is typically linear, on which may be superimposed an oscillation. The oscillation can cause observable non-gaussianity of the curvature perturbation, while retaining a practically smooth spectrum [S. Hannestad, T. Haugbolle, P. R. Jarnhus, M. S. Sloth, JCAP **1006**, 001 (2010)].

****Trapped inflation*

‘Trapped inflation’ means that the inflaton is continually dumping energy into the production of particles. In contrast with warm inflation, the particles do not thermalize. Observable non-gaussianity of the curvature perturbation can easily be produced. (D. Green, B. Horn, L. Senatore and E. Silverstein, Phys. Rev. D **80** (2009) 063533.)

**** GUT hybrid inflation without the gauge singlet*

As mentioned in Section 28.7, GUT hybrid inflation is well-motivated and can be part of a rather complete model of the early universe. The inflaton is usually taken to be a gauge singlet introduced only to give inflation. But it might instead have a gauge coupling, and play an important role in the post-inflation history. An example has been given in which the inflaton is a right-handed sneutrino; S. Antusch, M. Bastero-Gil, J. P. Baumann, K. Dutta, S. F. King and P. M. Kostka, JHEP **1008** (2010) 100.

**** Peccei-Quinn hybrid inflation*

Instead of invoking GUT symmetry breaking to generate the hybrid inflation potential, one can invoke Peccei-Quinn symmetry breaking, using the same superpotential [M. Kawasaki, N. Kitajima and K. Nakayama, arXiv:1104.1262]. Assuming as usual that the inflaton perturbation generates the curvature perturbation, this requires $f_{\rm a} \sim 10^{15}\,$GeV which is allowed within the model because the axion density is diluted by entropy production. One can also invoked a superpotential which is the sum of the GUT and Peccei-Quinn superpotentials, allowing the usual $f_{\rm a} \sim 10^{12}\,$GeV [G. Lazarides and C. Pallis, Phys. Rev. D **82** (2010) 063535].

**** Reheating after MSSM inflation*

If the inflaton potential is given by (28.17), using a flat direction of the MSSM, the course of the subsequent preheating and reheating is known once the flat direction is specified along with the relevant parameters of the MSSM. This has been done for a particular case (R. Allahverdi, A. Ferrantelli, J. Garcia-Bellido and A. Mazumdar, arXiv:1103.2123 [hep-ph]), leading to a reheat temperature of order $10^8\,$GeV.

**** Axionic coupling of the inflaton to a gauge field*

A scalar field ϕ can have a coupling to a gauge field A_μ of the form

$$\mathcal{L} \supset -\frac{\phi}{f} F_{\mu\nu} \tilde F^{\mu\nu},$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde F^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$. This ‘axionic’ coupling can generate a periodic potential, which with ϕ the inflaton might support Natural Inflation. But even if it is not responsible for the inflaton potential, an axionic coupling of the inflaton will cause the creation of A_μ from the vacuum. This field will affect $\delta\phi$ and hence the curvature perturbation $\zeta = -H\delta\phi/\dot\phi$, leading to possibly observable non-gaussianity (N. Barnaby and R. Namba and M. Peloso, arXiv:1102.4333). It will also generate primordial gravitational waves, with different spectra for R and L modes that might be visible in CMB anisotropy (L. Sorbo, arXiv:1101.1525 [astro-ph.CO].)

Miscellaneous developments

*** *Inflation with fields whose mass is of order H*

As stated in Section 20.2.2, we assume that all relevant fields during observable inflation are either heavy or light, ie we assume that no relevant field has effective mass of order H. That is the usual attitude, but recently the latter case is considered by X. Chen and Y. Wang, JCAP **1004** (2010) 027.

****Early-universe structure formation from an oscillating scalar field*

Even if the usual decay and preheating mechanisms are negligible, an oscillating scalar field will not last forever because the initial density perturbation (corresponding to the primordial density perturbation) will grow. This growth is more complicated than would be the case for a pressureless gas and can lead to a variety of observational consequences including the formation of mini-black holes (that typically would decay leading to reheating) and the generation of primordial gravitational waves. (R. Easther, R. Flauger and J. B. Gilmore, arXiv:1003.3011 [astro-ph.CO].)

*** *Gravitational wave production from the curvature perturbation*

As noted in Section 24.7.3, primordial gravitational waves can be created in the early universe by a variety of mechanisms. A mechanism that we didn't mention is their creation from the primordial density perturbation, generated as each scale enters the horizon by the primordial curvature perturbation ζ. The strength of such gravitational waves is limited by the black hole bound $\mathcal{P}_\zeta \lesssim 10^{-2}$ mentioned in Section 23.4, but that still allows future gravitational wave detectors to find them, or else place tighter upper limits on. (H. Assadullahi and D. Wands, Phys. Rev. D **81** (2010) 023527].)

*** *Black hole bound on* $\mathcal{P}_\zeta(k)$

On page 377 we note that cosmological constraints on the abundance of primordial black holes can place an upper bound on the spectrum $\mathcal{P}_\zeta(k)$ of the curvature perturbation at horizon entry. To obtain the bound, one assumes that the spatial average of ζ^2 when a given scale k enters the horizon will be at least of order $\mathcal{P}_\zeta(k)$. From Eq. (6.22), this will hold unless $\mathcal{P}(k)$ has a narrow peak at the given k, which is not expected within the inflationary cosmology. (Also, a narrow peak would require a different discussion because the exact choice $k = aH$ to define horizon entry is just a convention.)

But these calculations also assume that ζ is roughly $\lesssim 1$ everywhere in space, which is justified only if $\mathcal{P}_\zeta(k)$ is not much bigger than 1. In the opposite case there will be regions with $\zeta \gg 1$. In such a region, the spatial geometry is strongly distorted so that the wavenumber k, defined in the background, no longer corresponds to the physical size of the perturbation in the actual universe (M. Kopp, S. Hofmann, J. Weller, [arXiv:1012.4369 [astro-ph.CO]). One may reasonably conjecture that the case $\mathcal{P}_\zeta(k) \gg 1$ will still lead to excessive black hole formation, but there is no proof of that at present.